

# Automated Assume-Guarantee Reasoning for Omega-Regular Systems and Specifications

Software Engineering Institute  
Carnegie Mellon University  
Pittsburgh, PA 15213

*Arie Gurfinkel* and Sagar Chaki

April 13, 2010

Second NASA Formal Methods Symposium



## NO WARRANTY

THIS CARNEGIE MELLON UNIVERSITY AND SOFTWARE ENGINEERING INSTITUTE MATERIAL IS FURNISHED ON AN "AS-IS" BASIS. CARNEGIE MELLON UNIVERSITY MAKES NO WARRANTIES OF ANY KIND, EITHER EXPRESSED OR IMPLIED, AS TO ANY MATTER INCLUDING, BUT NOT LIMITED TO, WARRANTY OF FITNESS FOR PURPOSE OR MERCHANTABILITY, EXCLUSIVITY, OR RESULTS OBTAINED FROM USE OF THE MATERIAL. CARNEGIE MELLON UNIVERSITY DOES NOT MAKE ANY WARRANTY OF ANY KIND WITH RESPECT TO FREEDOM FROM PATENT, TRADEMARK, OR COPYRIGHT INFRINGEMENT.

Use of any trademarks in this presentation is not intended in any way to infringe on the rights of the trademark holder.

This Presentation may be reproduced in its entirety, without modification, and freely distributed in written or electronic form without requesting formal permission. Permission is required for any other use. Requests for permission should be directed to the Software Engineering Institute at [permission@sei.cmu.edu](mailto:permission@sei.cmu.edu).

This work was created in the performance of Federal Government Contract Number FA8721-05-C-0003 with Carnegie Mellon University for the operation of the Software Engineering Institute, a federally funded research and development center. The Government of the United States has a royalty-free government-purpose license to use, duplicate, or disclose the work, in whole or in part and in any manner, and to have or permit others to do so, for government purposes pursuant to the copyright license under the clause at 252.227-7013.



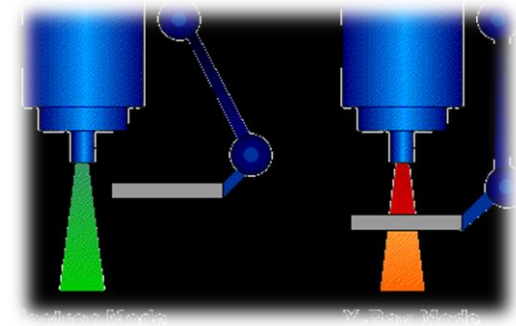
# When Failure is Not an Option

## Failure is not an option for

- Safety Critical Systems (e.g., X-rays machines)
- Medical Devices (e.g., infusion pumps, ...)
- Embedded Software (e.g., cars, airplanes)
- Security Vulnerabilities (e.g., nuclear plants)
- ...

## Formal software verification is essential to guarantee absence of failures

- **Automated** techniques include model checking and static analysis ...
- which can be used to validate, for example, safety and security, behavior **prior** to program execution
- and provide objective **evidence** of safe behavior



# Assume Guarantee Reasoning

System  $M_1$  in parallel with system  $M_2$  satisfies specification  $S$   
iff there exists an *assumption*  $A$  such that

- $M_1$  in parallel with  $A$  satisfies  $S$
- $M_2$  satisfies the assumption  $A$

$$\frac{M_1 \parallel A \models S \quad M_2 \models A}{M_1 \parallel M_2 \models S}$$

How to *automatically* find a sufficiently good assumption?!



# Related Work

## Safety properties

- Giannakopoulou et al. ASE 2002 – computing weakest assumption
- Cobliegh et al. TACAS 2003 – using  $L^*$  to learn “good-enough” assumption
- Barringer et al. SAVCBS 2003 – AG proof rules, soundness, completeness
- many follow up works to improve algorithms, complexity, applicability, etc.

## Liveness properties

- Farzan et al. TACAS 2008 –  $L^\$$  a learning algorithm for omega-regular languages

- **THIS PAPER**



- Assume-Guarantee proof rules for reactive (omega-regular) systems
- Soundness and (in)completeness
- Two new learning algorithms for infinitary (finite + infinite) languages
- A unifying framework for learning-based automated AG (see paper)



# Outline

## Background

- Model of concurrency: LTS, composition, specifications, etc.
- Active learning of regular languages:  $L^*$
- Learning-based Automated Assume Guarantee Framework

## Non-Circular Assume Guarantee Rule (AG-NC)

- Soundness and *in-completeness* for omega-regular languages
- Soundness and *completeness* for  $\infty$ -regular languages
- Learning algorithms for  $\infty$ -regular languages

## Circular Assume Guarantee Rule (AG-C)

- Soundness and completeness

## Conclusion and Future Work



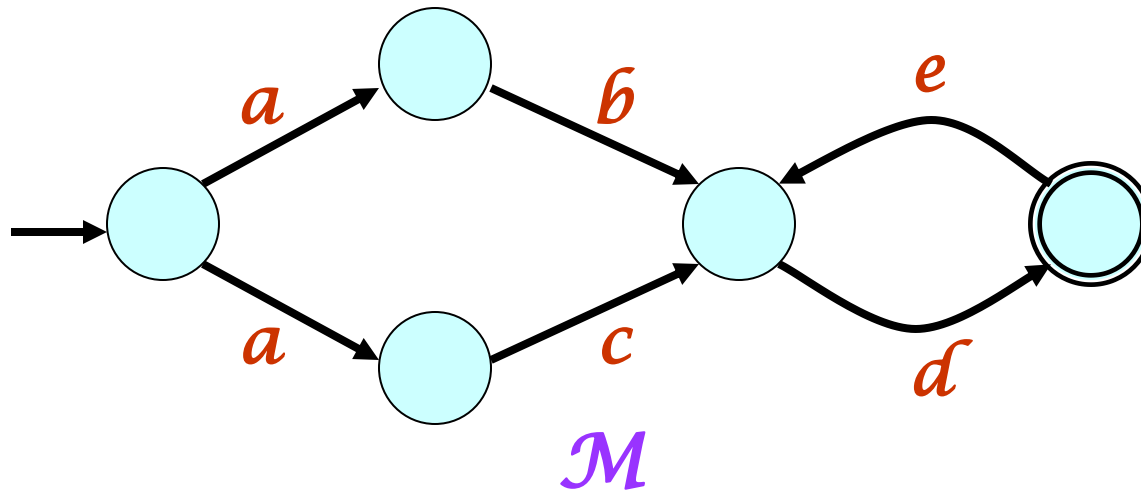
# Labeled Transition System (LTS)

$$M = (Q, I, \Sigma, T)$$

- $Q$  -- non-empty set of states
- $I \in Q$  an initial state
- $\Sigma$  -- set of actions (a.k.a, the alphabet)
- $T \subseteq Q \times \Sigma \times Q$  – a transition relation

+

**FA or Buchi  
acceptance condition**



$$\Sigma(\mathcal{M}) = \{a, b, c, d, e, f\}$$



# Operational Semantics

## CSP Semantics

- handshake (synchronize) over shared actions
- otherwise, proceed independently (asynchronously)

Composition  $M_1 \parallel M_2$  is

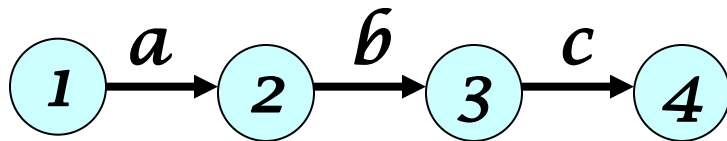
- State of  $M_1 \parallel M_2$  is of the form  $(s_1, s_2)$ , where  $s_i$  is a state of  $M_i$

$$\begin{array}{c}
 \frac{s_1 \xrightarrow{a} t_1 \quad a \notin \Sigma(\mathcal{M}_2)}{(s_1, s_2) \xrightarrow{a} (t_1, s_2)} \qquad \frac{s_2 \xrightarrow{a} t_2 \quad a \notin \Sigma(\mathcal{M}_1)}{(s_1, s_2) \xrightarrow{a} (s_1, t_2)} \\
 \\
 \frac{s_1 \xrightarrow{a} t_1 \quad s_2 \xrightarrow{a} t_2}{(s_1, s_2) \xrightarrow{a} (t_1, t_2)}
 \end{array}$$

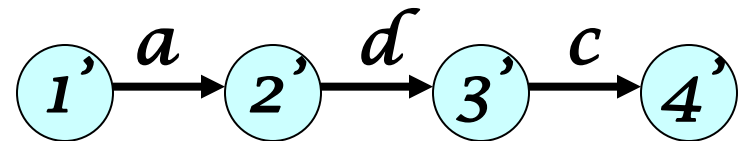




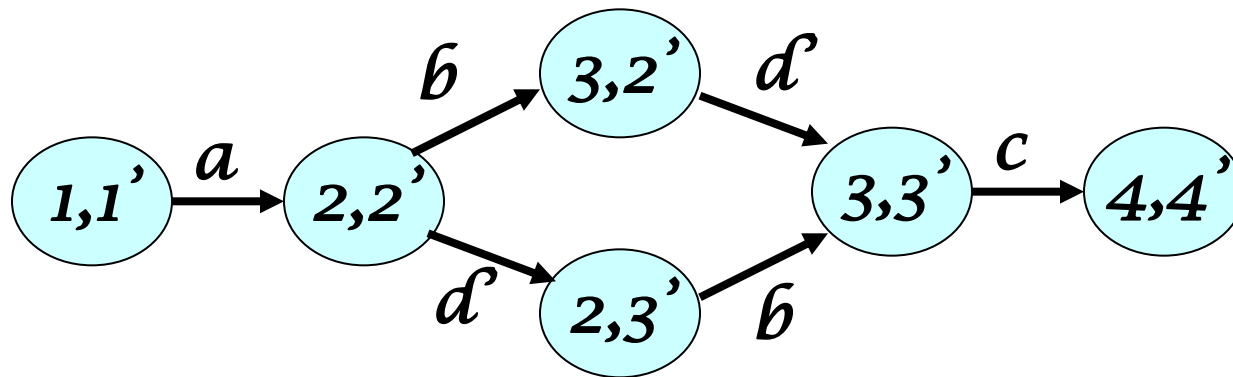
# Example of Composition



$\mathcal{M}_1 \quad \Sigma = \{a, b, c\}$



$\mathcal{M}_2 \quad \Sigma = \{a, d, c\}$



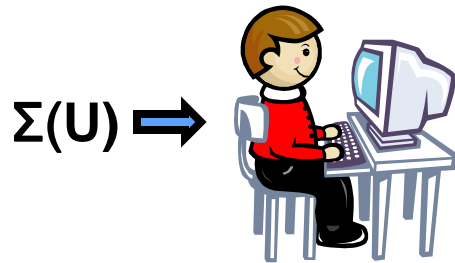
$\mathcal{M}_1 \parallel \mathcal{M}_2 \quad \Sigma = \{a, b, d, c\}$



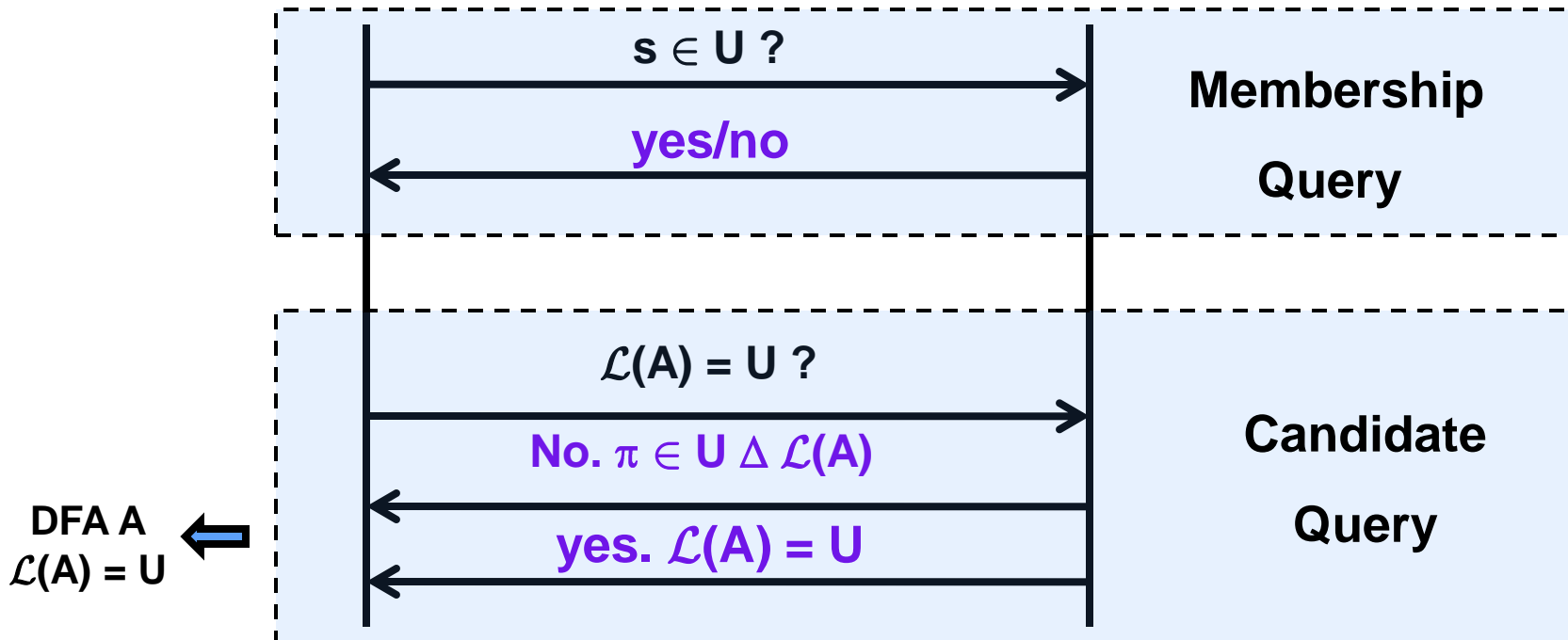
# $L^*$ (Angluin 1987, Rivest & Schapire 1993)

$L^*$  learner

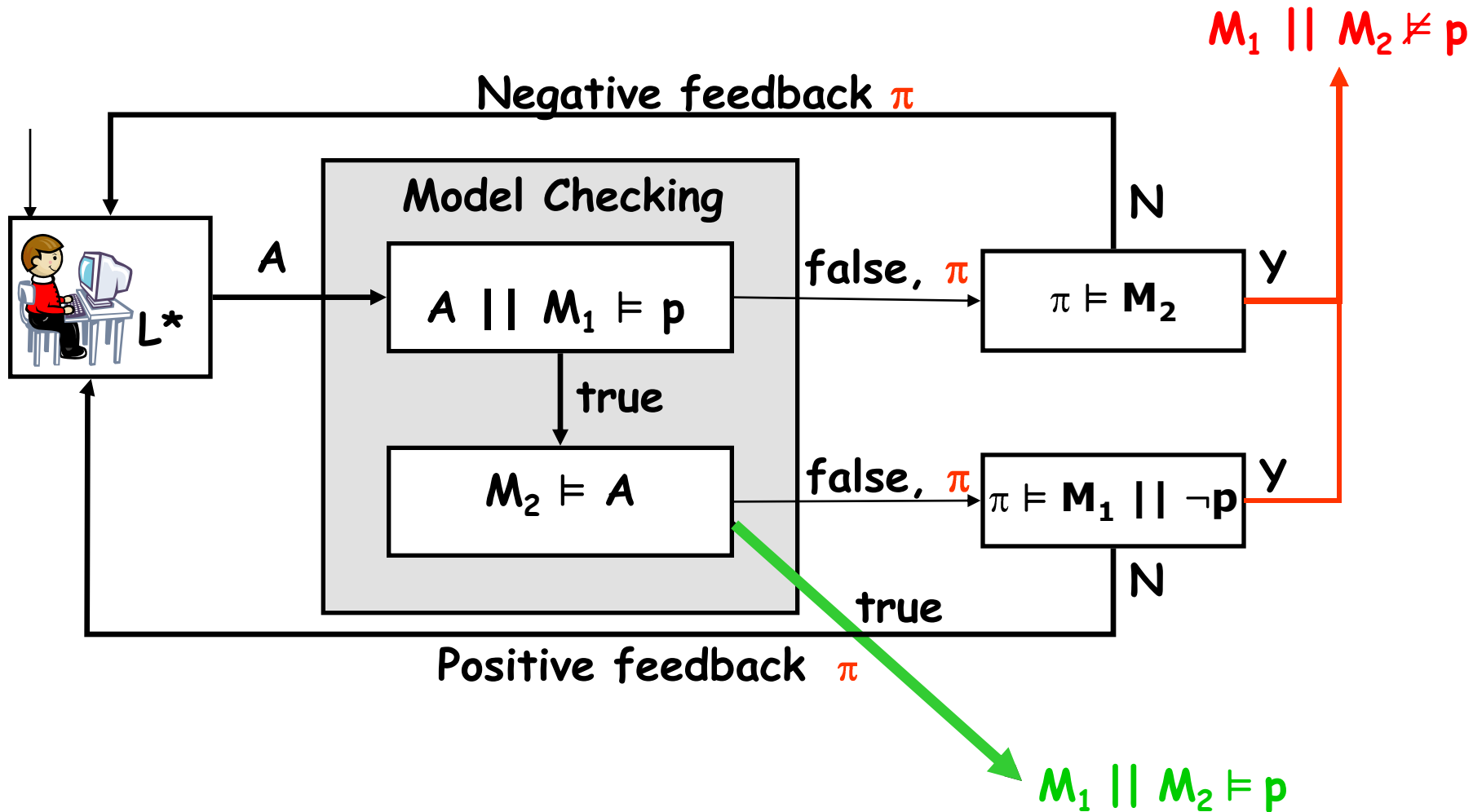
Minimally Adequate Teacher (MAT)



$U$  -- regular language



# Assume Guarantee with Learning



# Outline

## Background

- Model of concurrency: LTS, composition, specifications, etc.
- Active learning of regular languages:  $L^*$
- Learning-based Automated Assume Guarantee Framework

## ➡ Non-Circular Assume Guarantee Rule (AG-NC)

- Soundness and *in-completeness* for omega-regular languages
- Soundness and *completeness* for  $\infty$ -regular languages
- Learning algorithms for  $\infty$ -regular languages

## Circular Assume Guarantee Rule (AG-C)

- Soundness and completeness

## Conclusion and Future Work



# AG-NC: Non-Circular Assume Guarantee Rule

$$L \preceq S \text{ iff } L \downarrow \Sigma_S \subseteq S$$

$$(L_1 \parallel L_A) \preceq L_S \quad L_2 \preceq L_A$$

---

$$(L_1 \parallel L_2) \preceq L_S$$

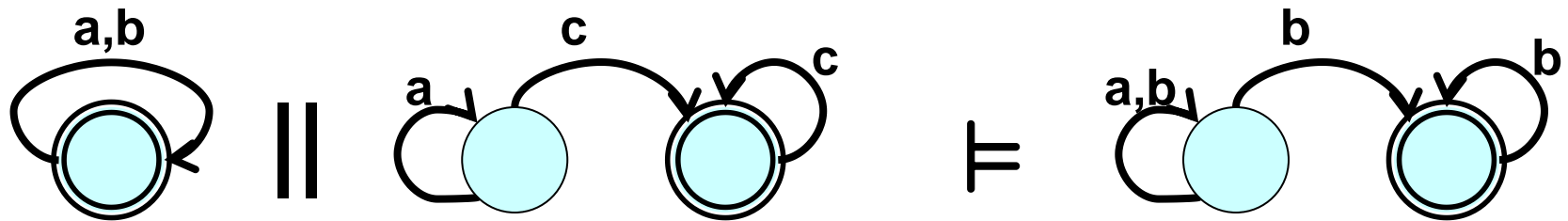
$$\Sigma_A = (\Sigma_1 \cup \Sigma_S) \cap \Sigma_2$$

Complete for Safety (regular) properties ( $\Sigma^*$ )

Incomplete for Liveness (omega-regular) properties ( $\Sigma^\omega$ )



# Proof of Incompleteness (by Counterexample)



$$\Sigma_1 = \{a, b\}$$

$$\Sigma_2 = \{a, c\}$$

$$\Sigma_S = \{a, b\}$$

$$L_1 = (a+b)^\omega$$

$$L_2 = a^*c^\omega$$

$$L_S = (a+b)^*b^\omega$$

Assumption alphabet:  $\Sigma_A = \{a\}$

**BUT, there is no assumption  $L_A \subseteq \Sigma_A^\omega$  to apply AG-NC**

$$A_1 = \emptyset \quad L_2 \not\leq A_1$$

$$A_2 = a^\omega \quad L_1 \parallel A_2 \not\leq L_S$$



# AG-NC: Infinite Trace Containment

$$L \preceq_{\omega} S \text{ iff } \omega(L \downarrow \Sigma_S) \subseteq \omega(S)$$

$$(L_1 \parallel L_A) \preceq_{\omega} L_S$$

$$L_2 \preceq_{\omega} L_A$$

$$\Sigma_A = (\Sigma_1 \cup \Sigma_S) \cap \Sigma_2$$

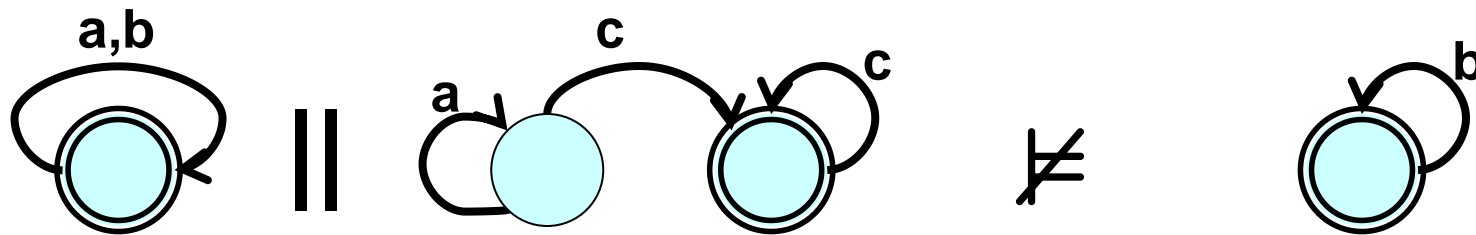
---

$$(L_1 \parallel L_2) \preceq_{\omega} L_S$$

**NOT SOUND!**



# Proof of Unsoundness (by Counterexample)



$$\Sigma_1 = \{a, b\}$$

$$\Sigma_2 = \{a, c\}$$

$$\Sigma_S = \{a, b\}$$

$$L_1 = (a+b)^\omega$$

$$L_2 = a^*c^\omega$$

$$L_S = b^\omega$$

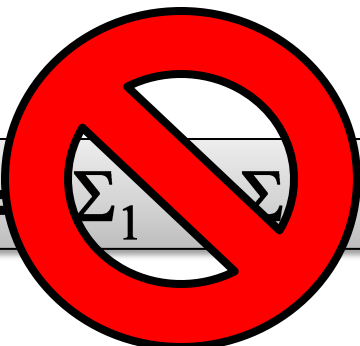
$$\Sigma_A = \{a\}$$

**BUT,  $L_A = \emptyset$  satisfies all premises of (modified) AG-NC**





# AG-NC: Relaxing Assumption Alphabet

$$\frac{(L_1 \parallel L_A) \preceq L_S \quad L_2 \preceq L_A}{(L_1 \parallel L_2) \preceq L_S} \quad \Sigma_A = \Sigma_1 \cap \Sigma_2$$


Complete for Safety (regular) properties ( $\Sigma^*$ )

Complete for Liveness (omega-regular) properties ( $\Sigma^\omega$ )

Assumption “knows” about internal actions of  $L_1$  and  $L_2$

Not “truly” compositional



# AG-NC: Restoring Completeness

**Theorem:** Let  $L_1$  and  $L_S$  be two languages, and  $\Sigma_A$  an alphabet s.t.  $\Sigma_1 \cup \Sigma_A = \Sigma_1 \cup \Sigma_S$ . Then,  $L_A = \mathcal{C}((L_1 \parallel \mathcal{C}(L_S)) \downarrow \Sigma_A)$  is the *weakest* assumption such that  $L_1 \parallel L_A \preceq L_S$

## Corollaries:

**AG-NC is complete for any class of languages closed under *projection* and *complement***

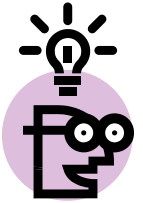
**AG-NC is complete for  $\Sigma^*$**

**AG-NC is complete for  $\Sigma^\infty = \Sigma^* \cup \Sigma^\omega$**

**Need a learning algorithm for  $\Sigma^\infty$ !**



# Learning Infinitary Language U: Approach 1



Use  $L^*$  and  $L^\$$  *simultaneously* to learn a DFA  $D$  and a BA  $B$  such that  
 $\mathcal{L}(\text{DFA}) = {}^*(U)$  and  $\mathcal{L}(\text{BA}) = \omega(U)$

Assume  $M$  is a MAT for  $U$

Use  $M$  to answer membership queries until *both* learners generate a candidate query

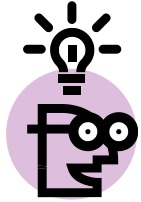
Use  $M$  to verify the candidate query:  $\mathcal{L}(D) \cup \mathcal{L}(B) = U$

- on success, stop
- if counterexample is finite, send to  $L^*$  and resume until next DFA candidate
- if counterexample is infinite, send to  $L^\$$  and resume until next BA candidate

**Two learners. Possibly a lot of redundancy.**



# Learning Infinitary Language U: Approach 2



Use  $L^\$$  to learn  $U.\tau^\omega$ , where  $\tau$  is a “fresh” symbol not in  $\Sigma_U$

Assume M is a MAT for U

To answer a membership query infinite word s

- if  $s = t.\tau^\omega$  and  $t \in \Sigma_U^\infty$  then ask M whether  $t \in U$  and forward answer back
- Otherwise, answer “no”

To answer a candidate query with candidate BA C

- if  $\mathcal{L}(C) \not\subseteq \Sigma_U^\infty.\tau^\omega$  return  $\pi \in \mathcal{L}(C) \setminus \Sigma_U^\infty.\tau^\omega$
- otherwise, forward candidate query  $*(\mathcal{L}(A)\downarrow\Sigma), \omega(\mathcal{L}(A)\downarrow\Sigma)$  to M

**Single learner, BUT larger alphabet**



# Circular AG-rule (AG-C): Summary

$$\Sigma_{A1} = \Sigma_{A2} = (\Sigma_1 \cap \Sigma_2) \cup \Sigma_S$$

$$\frac{L_1 \parallel L_{A1} \preceq L_S \quad L_2 \parallel L_{A2} \preceq L_S \quad \mathcal{C}(L_{A1}) \parallel \mathcal{C}(L_{A2}) \preceq L_S}{L_1 \parallel L_2 \preceq L_S}$$

Complete for Safety (regular) properties ( $\Sigma^\infty$ )

Complete for Liveness (omega-regular) properties ( $\Sigma^\omega$ )



**BUT need to learn 2 assumptions AND assumption alphabet is larger**



# Learning-Based AG (LAG) Framework

Conformance	Rule	$\mathcal{A}$	Learner(s)	Oracle(s)	Checker
Regular Trace Containment	<b>AG-NC</b> [1]	DFA	$P_1 = P(\mathbf{L}^*)$	$Q_1 = Q(L_1, L_S, \Sigma_{NC})$	$V_{NC}(L_1, L_2, L_S)$
Regular Trace Containment	<b>AG-C</b> [2]	DFA	$P_1 = P_2 = P(\mathbf{L}^*)$	$Q_1 = Q(L_1, L_S, \Sigma_C)$ $Q_2 = Q(L_2, L_S, \Sigma_C)$	$V_C(L_1, L_2, L_S)$
$\infty$ -regular Trace Containment	<b>AG-NC</b>	DFA $\times$ BA	$P_1 = P(\mathbf{L})$	$Q_1 = Q(L_1, L_S, \Sigma_{NC})$	$V_{NC}(L_1, L_2, L_S)$
$\infty$ -regular Trace Containment	<b>AG-C</b>	DFA $\times$ BA	$P_1 = P_2 = P(\mathbf{L})$	$Q_1 = Q(L_1, L_S, \Sigma_C)$ $Q_2 = Q(L_2, L_S, \Sigma_C)$	$V_C(L_1, L_2, L_S)$
$\omega$ -regular Trace Containment	<b>AG-NC</b>	DFA $\times$ BA	$P_1 = P(\mathbf{L})$	$Q_1 = Q(L_1, L_S, \Sigma_{NC})$	$V_{NC}(L_1, L_2, L_S)$
$\omega$ -regular Trace Containment	<b>AG-C</b>	BA	$P_1 = P_2 = P(\mathbf{L}^\omega)$	$Q_1 = Q(L_1, L_S, \Sigma_C)$ $Q_2 = Q(L_2, L_S, \Sigma_C)$	$V_C(L_1, L_2, L_S)$

[1] Cobleigh, Giannakopoulou, Pasareanu, TACAS'03

[2] Barringer, Giannakopoulou, Pasareanu, SAVCBS'03

The last four rows are contributions of THIS PAPER



# Conclusion and Future Work

Compositional approach to verification is fundamental for scalability!

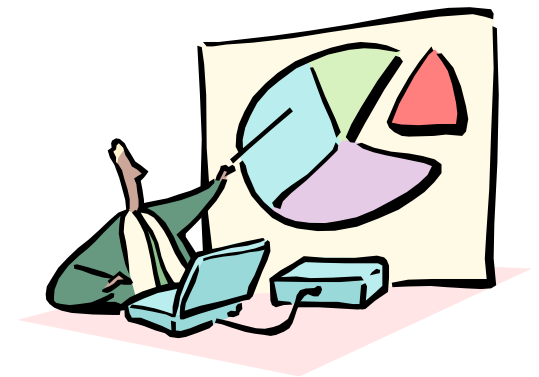
Automated AG for Liveness (omega-regular) properties

- Non-Circular Rule: soundness, (in)completeness
- Circular rule – remains sound and complete
- Two new learning algorithms for infinitary languages

Unified Framework for Learning-based Assume Guarantee Reasoning

Future Work

- implementation and empirical evaluation
- experiments with other learning algorithms



# THE END





# Contact Information

## Presenter

Arie Gurfinkel

RTSS

Telephone: +1 412-268-5800

Email: [arie@cmu.edu](mailto:arie@cmu.edu)

## Web:

[www.sei.cmu.edu](http://www.sei.cmu.edu)

<http://www.sei.cmu.edu/contact.cfm>

## U.S. mail:

Software Engineering Institute

Customer Relations

4500 Fifth Avenue

Pittsburgh, PA 15213-2612

USA

## Customer Relations

Email: [info@sei.cmu.edu](mailto:info@sei.cmu.edu)

Telephone: +1 412-268-5800

SEI Phone: +1 412-268-5800

SEI Fax: +1 412-268-6257

